

# Cosmic Acceleration Driven by Mirage Inhomogeneities

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**Abstract.** A cosmological model based on an inhomogeneous D3-brane moving in an  $AdS_5 \times S_5$  bulk is introduced. Although there is no special points in the bulk, the brane Universe has a center and is isotropic around it. The model has an accelerating expansion and its effective cosmological constant is inversely proportional to the distance from the center, giving a possible geometrical origin for the smallness of a present-day cosmological constant. Besides, if our model is considered as an alternative of early time acceleration, it is shown that the early stage accelerating phase ends in a dust dominated FRW homogeneous Universe. Mirage-driven acceleration thus provides a dark matter component for the brane Universe final state. We finally show that the model fulfills the current constraints on inhomogeneities.

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## 1. Introduction

The motion of a D-brane in some gravitational background follows a classical geodesic, so its induced metric is time dependent. The motion of the brane leads to a time-dependent geometry, which is interpreted as a cosmological evolution by a brane-observer. This has been called mirage cosmology [1] and technically, there are two steps in determining the resulting cosmological evolution. To start with, the brane motion is determined by

solving the world-volume field equations, as follow from the Dirac-Born-Infeld (DBI) action, for the scalar fields which describe the position of the brane in the bulk. Then, the induced metric on the brane is specified, now becoming an implicit function of time, giving a cosmological evolution for the brane. This cosmological evolution can be reinterpreted in terms of cosmological “mirage” energy densities on the brane via a Friedman-like equation.

Various combinations of probe branes on simple backgrounds have been analyzed [1], such as stacks of Dp-branes at and out of extremality (black Dp-branes), with and without additional constant antisymmetric background tensors. They provide a cosmological evolution on the probe brane which can be simulated by various types of mirage matter on the brane. There is no a priori reason for the mirage energy-momentum tensor to satisfy the energy conditions as it does not describe real matter on the brane. What drives the cosmological evolution is not some form of matter or energy, but the brane motion in the background geometry. Indeed, in general, for a Dp-brane moving in the background of other Dp'-branes, expansion on the world-volume of the probe brane was found to correspond to an equation of state for the mirage pressure and energy of the form  $p = w\rho$  with  $w > 1$ , violated by all forms of matter. It should be stressed however that there are many exotic types of mirage matter including  $w$  values that are outside the range  $|w| \leq 1$  required by four-dimensional causality.

Another peculiarity of this framework is that individual densities of dilute mirage matter can be negative (without spoiling the overall positivity at late times). This is linked to the fact that in this type of cosmology, the initial singularity is an artifact of the low energy description. This can be seen, for example, by studying brane motion in simple spaces like  $AdS_5 \times S^5$  which are globally non-singular. The induced cosmological evolution of a brane moving in such a space has a typical expansion profile due to radiation and an initial singularity (from the four-dimensional point of view). At the initial singularity the brane Universe joins a collection of parallel similar branes and there is (non-abelian) symmetry enhancement. The effective field theory breaks down and this gives rise to the singularity.

The next obvious question is how “real” matter/energy densities on the brane affect its geodesic motion and the induced cosmological evolution. This has been studied in the mirage-cosmology context by turning on electromagnetic energy on the brane. An important point here is that the cosmological evolution is not driven by four-dimensional gravity on the brane.

Motivated by the homogeneity of the D-brane backgrounds, (see however [2]), a silent feature of the mirage-cosmology scenario is that the resulting brane-cosmology is isotropic and homogeneous. This is due to the fact that the scalars which describe the position of the brane in the bulk geometry are taken to only be time-dependent. This corresponds to a rigidly moving three-brane in the bulk, and to our knowledge, on such backgrounds, the mirage matter alone cannot account for the acceleration of the Universe’s expansion [3]‡. We here consider a brane moving in such a way that its

‡ In order to obtain an accelerating Universe in this scenario, it is indeed necessary to turn on a

motion depends not only on time but also on some other space directions. In this case, the induced metric on the brane is neither isotropic nor homogeneous. Inhomogeneous cosmologies are well studied in the literature, and it has been shown that the current data cannot rule them out (see [8] for a review). Moreover, it has been suggested in [9] that late time cosmology cannot be approximately homogeneous. However, the recent proposals that inhomogeneities in a matter dominated Universe, on either super-horizon [6] or sub-horizon scales [7] may influence the expansion rate at late times have been criticized [7],[10],[11],[12]. In fact, it has been shown that sub-horizon [11] and super-horizon [7] perturbations,[10],[12] are not a viable candidate for explaining the accelerated expansion of the Universe. This is simply because as long as energy conditions are valid, inhomogeneous models can never lead to accelerated expansion [12]. It should be noted that inhomogeneous cosmological models in the context of string theory have previously been discussed [14].

Here, however, we will show that in mirage cosmology, inhomogeneities may indeed lead to accelerated cosmic expansion either at early or late times. In particular, inhomogeneities compatible with  $SO(3)$  symmetry may lead to an accelerating expansion phase for a probe D3-brane moving in the near horizon limit of a background of a stack of D3-branes. When this phase happens at early times, it may provide a mechanism for inflation. In that case, exit of the inflation is achieved when inhomogeneities are diluted enough due to inflation. We will see that what remains of these inhomogeneities is similar to pressureless matter. Thus, in this scenario, mirage matter left over at the end of the inflationary phase, may be a dark matter component. On the other hand, when the accelerating phase is at late times, mirage cosmology can provide an alternative to  $\Lambda$ CDM.

It should also be noted that our model does not have any initial singularity so that it naturally fits the homogeneity of the CMB spectra. In fact, in the absence of big bang-like singularities, all parts of the universe may have been in causal contact in the far past. Moreover we also show that the observed small scale homogeneities (in the range of  $100 \div 140$  Mpc [17]) can be implemented in our model and related to the best fit cosmological constant of [5].

## 2. Set-up

In general, a probe D-brane light enough for its back-reaction to be neglected will follow a geodesic as it moves in some string-theory background. Due to its motion, the induced world-volume metric on the probe brane becomes a function of time so that, from the brane “residents” point of view, a changing (expanding or contracting) Universe is experienced. This is the basic idea behind mirage cosmology. The simplest case corresponds to a D3-brane moving in a static spherically symmetric gravitational background with a dilaton as well as a RR background  $C(r) = C_{0\dots 3}(r)$  with a self-dual tachyonic bulk field. In this case it is also possible to have a mechanism for the reheating of the Universe [4].

field strength. The probe brane will in general start moving in this background along a geodesic and its dynamics is governed by the DBI action. In the case of maximal supersymmetry the DBI action is

$$S = T_3 \int d^4 \xi e^{-\phi} \sqrt{-\det(\hat{G}_{\alpha\beta} + (2\pi\alpha')F_{\alpha\beta} - B_{\alpha\beta})} + T_3 \int d^4 \xi \hat{C}_4 + \text{anomaly term} \quad (1)$$

where world-volume fermions have been ignored and  $\xi^\alpha$  ( $\alpha = 0, 1, 2, 3$ ) parameterize the D3-brane world-volume. The embedded data are given by

$$\hat{G}_{\alpha\beta} = G_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta}, \quad (2)$$

etc. For simplicity we will assume that the  $B$ -field is zero and there is no electromagnetic field on the brane.

In particular, we will consider here a probe D3-brane moving in the near-horizon limit of a stack of D3-branes with metric

$$ds^2 = \frac{r^2}{L^2} (-dt^2 + d\rho^2 + \rho^2 d\Omega_2^2) + \frac{L^2}{r^2} dr^2 + L^2 d\Omega_5^2, \quad (3)$$

where we have chosen spherical coordinates for the brane directions. This is just an  $AdS_5 \times S^5$  space-time and  $L$  is the  $AdS_5$  length defined by  $L^4 = 4\pi g_s N$ , where  $g_s$  is the string coupling and  $N$  the D3-brane charge. The RR field is

$$C_{0..3} = \frac{r^4}{L^4} \quad (4)$$

and the dilaton  $\phi$  is constant. We will regard our Universe as a D3-brane moving in this background.

We now consider an inhomogeneous trajectory for the brane motion  $r = r(t, \rho)$ . The case of homogeneous trajectory has been studied before [1]. No matter is included on the brane and only geometrical quantities related to the chosen embedding will play a role. The induced metric is then

$$ds_{ind}^2 = - \left( \frac{r^2}{L^2} - \frac{\dot{r}^2 L^2}{r^2} \right) dt^2 + \left( \frac{r^2}{L^2} + \frac{r'^2 L^2}{r^2} \right) d\rho^2 + 2 \frac{\dot{r} r'}{r^2} L^2 d\rho dt + \frac{r^2}{L^2} \rho^2 d\Omega_5^2,$$

where  $r = r(t, \rho)$  and a dot denotes derivative with respect to time ( $\dot{\phantom{x}} = \partial_t$ ), whereas a prime denotes derivative with respect to brane-radial distance ( $\phantom{x}' = \partial_\rho$ ).

Due to re-parametrization invariance, there is a gauge freedom, which may be fixed by choosing the static gauge,  $x^\alpha = \xi^\alpha$ , so that in this gauge, the total Lagrangian is

$$\frac{\mathcal{L}}{4\pi T_3} = \frac{r^4}{L^4} \rho^2 \sqrt{1 + \frac{r'^2 L^4}{r^4} - \frac{\dot{r}^2 L^4}{r^4}} - \frac{r^4}{L^4}. \quad (6)$$

In the following we will consider the “non-relativistic” approximations

$$\frac{r'^2 L^4}{r^4} \ll 1, \quad \frac{\dot{r}^2 L^4}{r^4} \ll 1. \quad (7)$$

In this case the field equations  $\delta\mathcal{L} = 0$  reduce to

$$\ddot{r} - \frac{1}{\rho^2} \partial_\rho (\rho^2 r') = 0. \quad (8)$$

As the bulk is regular, the only possible brane singularity happens when the brane shrinks to a point, i.e. when  $r = 0$ . The most general solution without singularity is

$$r = \frac{\mu}{2}(t - t_0)^2 + \frac{\mu}{6}\rho^2 + r_0, \quad (9)$$

as one can check by calculating all the curvature invariants. Here  $r_0(> 0)$  corresponds to the position of the center of the brane at some fiducial time  $t_0$  and  $\mu$  is a mass-parameter. We also would like to stress that this solution does not have a homogeneous limit, and thus belongs to a new branch of solutions.

Note that the above discussion is valid in the non-relativistic approximation (7). Using (9), it is easy to verify that a sufficient condition for the validity of this approximation is

$$\mu \ll \frac{r_0^3}{L^4}. \quad (10)$$

### 3. Accelerating Universe

In this section, we will show that the expansion rate of geodesic congruences is accelerated with a natural ending, the end state corresponding to pressureless mirage matter. Depending on when mirage effects dominate conventional Einstein gravity, inhomogeneous mirage matter may lead to either early or late time cosmological acceleration. In particular, it may provide an alternative to inflation, or to the late-time cosmic expansion we observe today. We may employ the present model for inflationary cosmology if mirage matter is dominating at early times. Then, not only our Universe is accelerated at early time, but it does not have a cosmological singularity, avoiding the existence of a particle horizon. In our model, in fact, all parts of the Universe are connected to each other by a time-like curve and are separated by a finite amount of proper time. This does not happen for non-inflating singular cosmologies like the FRW model. In our case, therefore, the homogeneity of the CMB spectra is natural.

#### 3.1. Light-cone acceleration

To simplify the problem, we shall approximate the induced metric (5) by

$$ds_i^2 \simeq \frac{r^2}{L^2}(d\eta^2), \quad (11)$$

where  $d\eta^2$  is the four dimensional Minkowski metric. For our purpose, (11) will be a good approximation to (5) if the relative discrepancy of the Christoffel and the second variations of the determinant of the metrics are very small compared to 1 §. One can check that these conditions are met assuming the non-relativistic approximation (10).

§ i.e. if 'i' corresponds to the induced metric (5) and 'c' to the conformal one (11), we want  $\frac{{}^i\Gamma_{\beta\gamma}^\alpha - {}^c\Gamma_{\beta\gamma}^\alpha}{{}^i\Gamma_{\beta\gamma}^\alpha} \ll 1$  and similarly, we require that  $\frac{\partial^2 \sqrt{{}^i g} - \partial^2 \sqrt{{}^c g}}{\partial^2 \sqrt{{}^i g}} \ll 1$ , where  ${}^i g$  is the determinant of the corresponding metric

The four-velocity of a null radial geodesic in the metric (11) is then easily found to be

$$k^\alpha = \left[ -\frac{F(\rho+t)}{(\mu\rho^2 + 3\mu(t-t_0)^2 + 6r_0)^2}, \frac{F(\rho+t)}{(\mu\rho^2 + 3\mu(t-t_0)^2 + 6r_0)^2}, 0, 0 \right], \quad (12)$$

where  $F(\rho+t)$  is a non-determined function that depends on the parametrization chosen for the null geodesic.

Defining the expansion scalar as  $\theta = \nabla_\alpha k^\alpha$ , the acceleration parameter  $Q_n$  of a null-congruence is [13]

$$Q_n = k^\alpha \nabla_\alpha \theta + \frac{1}{2}\theta^2 = 96\mu^2 F(\rho+t)^2 \frac{(t-t_0) \left[ \rho - (t-t_0) + \frac{r_0}{\mu(t-t_0)} \right]}{[\rho^2 + 3(t-t_0)^2 + 6r_0]^6}. \quad (13)$$

We recall that for a null geodesic vector  $k^\alpha$ , the following geometrical condition holds

$$k^\alpha \nabla_\alpha \theta + \frac{1}{2}\theta^2 = -R_{\alpha\beta} k^\alpha k^\beta + \omega^2 - \sigma^2, \quad (14)$$

where the vorticity  $\omega_{\alpha\beta}$  and shear  $\sigma_{\alpha\beta}$  are defined as

$$\omega_{\alpha\beta} = \nabla_{[\alpha} k_{\beta]}, \quad \sigma_{\alpha\beta} = \tilde{h}_{(\alpha}^\mu \tilde{h}_{\beta)}^\nu \nabla_\mu k_\nu - \frac{1}{2}\theta^2, \quad (15)$$

in terms of the two-metric  $\tilde{h}_{\alpha\beta}$ , which satisfies  $\tilde{h}_{\alpha\beta} k^\alpha = 0$ . One can show that the four velocity (12) has zero vorticity and shear for the lowest order induced metric (11). So in order to have acceleration ( $Q_n > 0$ ), the null energy conditions  $R_{\alpha\beta} k^\alpha k^\beta \geq 0$  must be violated. Indeed, for the metric (11),  $R_{\alpha\beta} k^\alpha k^\beta = -Q_n$  so that when the Universe's light-cone expansion accelerates, the null energy conditions are violated. However, since no physical matter has really been used, there is no violation of any fundamental physical law. There is no reason to expect mirage matter to satisfy any energy condition, since it does not corresponds to any form of real matter. This has also been stressed before [1], where mirage matter violating causality bounds was found to drive cosmological expansion.

It is now easy to see that the light cone is accelerating ( $Q_n > 0$ ) for  $\rho + \frac{r_0}{\mu(t-t_0)} > (t-t_0)$  and that the acceleration naturally ends when  $\rho + \frac{r_0}{\mu(t-t_0)} = t-t_0$ . Thus, the larger the radial distance, the longer the accelerating era.

We would like to stress that the acceleration is only indirectly due to inhomogeneities. In fact, as we showed, the only contribution to the Raychaudhuri equation (13) is coming from the negative mirage energy density of the brane inhomogeneities.

### 3.2. Time-like congruences

For a time-like congruence, the geodesic equation is  $u^\alpha \nabla_\alpha u^\beta = 0$ , where the vector  $u^\alpha$  is normalized

$$u^\alpha u_\alpha = -1. \quad (16)$$

For a radial motion, we have that  $u^\alpha = (u^t(t, \rho), u^\rho(t, \rho), 0, 0)$  and Eq. (16) is satisfied for

$$u^t = -\sqrt{\frac{L^2}{r^2} + (u^\rho)^2}. \quad (17)$$

As it is difficult to solve the above geodesic equation, some approximations must be used to get analytical results. Let us consider the following quasi-homogeneous congruence

$$u^\rho = v \ll L/r , \quad (18)$$

so that the time component of  $u^\alpha$  can be approximated by

$$u^t = -\frac{L}{r} \left[ 1 + \frac{r^2}{2L^2} v^2 + \mathcal{O}\left(\frac{r^4}{L^4} v^4\right) \right] . \quad (19)$$

The geodesic motion turns out to be given, to leading order, by a single equation for  $u^\rho$

$$\dot{v} = \frac{r' L}{r^2} , \quad (20)$$

and its solution with initial condition  $v(t = t_0, \rho) = 0$  is

$$v = \frac{\alpha(t - t_0)}{2\beta[(t - t_0)^2 + \beta]} + \frac{\alpha}{2\beta^{3/2}} \arctan\left(\frac{t - t_0}{\beta^{1/2}}\right) . \quad (21)$$

The parameters  $\alpha, \beta$  are given by

$$\alpha = \frac{4\rho L}{3\mu} , \quad \beta = \frac{\rho^2}{3} + \frac{2r_0}{\mu} , \quad (22)$$

in terms of which we may express the solution in Eq.(9) as

$$r = \frac{\mu}{2} \left( (t - t_0)^2 + \beta \right) . \quad (23)$$

Since, to us, the Universe looks homogeneous, isotropic and accelerated, we may also consider our position as being very far from its center. According to the condition (10), we can consider  $\mu$  small and/or  $r_0$  large so that  $\beta$  is large. In the  $\beta \gg (t - t_0)^2$  limit we have

$$v = \frac{\alpha(t - t_0)}{\beta^2} + \mathcal{O}\left(\frac{1}{\beta^3}\right) , \quad (24)$$

and the condition (18) is fulfilled for

$$(t - t_0) \ll \frac{\rho}{2} + \frac{3r_0}{\mu\rho} . \quad (25)$$

Considering units such that our position is at  $\rho > 1$  and assuming  $t/t_0 > 1$ , we find that a sufficient condition for the validity of our approximation actually is

$$\beta \gg (t - t_0)^2 . \quad (26)$$

We may now calculate the acceleration parameter  $Q_t$ . For time-like geodesics, it is

$$Q_t = \dot{\theta} + \frac{1}{3}\theta^2 .$$

In the large  $\beta$  limit, we find that  $Q_t$  is given by

$$Q_t = \frac{32L^2}{3\mu^2\beta^3} + \mathcal{O}\left(\frac{1}{\beta^4}\right) . \quad (27)$$

As for null geodesics, we find that these time-like geodesic congruences are accelerating ( $Q_t > 0$ ). Considering our position in the Universe to be  $\rho = \rho_0 \gg \sqrt{6r_0/\mu}$ , we have

$$Q_t \simeq \frac{288L^2}{\mu^2\rho_0^6} = \Lambda , \quad (28)$$

where  $\Lambda$  is the cosmological constant as measured today. Thus, our relative position with respect to the center is

$$\rho_0 \simeq \left( \frac{288L^2}{\mu^2\Lambda} \right)^{1/6}, \quad (29)$$

and therefore, the measured small cosmological constant is in principle compatible with a large distance of our position with respect to the center of the Universe. It should be stress that a fine-tuning of  $Q_t$  is needed in order to match the observed acceleration. However, in this case the smallness of the latter is geometrical and due to the large distance of our position from the center. Still there is a fine-tuning as we have to choose the parameters  $\mu_0, L$ , which nevertheless is not better than the usual fine-tuning of the cosmological constant. Clearly, the present model is not relevant for the cosmological constant problem arising from the contribution of supersymmetry breaking, phase transitions etc. in the vacuum energy. As usual, we assume that a mechanism exists which neutralize all these vacuum-energy sources leaving only the mirage contribution to account for the observed acceleration.

One may also verify that the vorticity  $\omega_{\alpha\beta}$  and the shear  $\sigma_{\alpha\beta}$  given in terms of the three metric  $h_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$  by

$$\omega_{\alpha\beta} = \nabla_{[\alpha} u_{\beta]}, \quad \sigma_{\alpha\beta} = h_{(\alpha}^\mu h_{\beta)}^\nu \nabla_\mu u_\nu - \frac{1}{3}\theta^2, \quad (30)$$

vanish to lower order and

$$\omega^2 = \omega_{\alpha\beta}\omega^{\alpha\beta} = \mathcal{O}\left(\frac{1}{\beta^4}\right), \quad \sigma^2 = \sigma_{\alpha\beta}\sigma^{\alpha\beta} = \mathcal{O}\left(\frac{1}{\beta^4}\right). \quad (31)$$

In addition, a short calculation of the energy conditions reveals that

$$R_{\alpha\beta}u^\alpha u^\beta = -\frac{32L^2}{3\mu^2\beta^3} + \mathcal{O}\left(\frac{1}{\beta^4}\right), \quad (32)$$

and clearly, the strong energy conditions  $R_{\alpha\beta}u^\alpha u^\beta \geq 0$  are violated, while the expansion accelerates. For the time-like geodesic congruence we are discussing, the Raychaudhuri equation is satisfied,

$$\dot{\theta} + \frac{1}{3}\theta^2 = -R_{\alpha\beta}u^\alpha u^\beta + \omega^2 - \sigma^2. \quad (33)$$

A comparison to the FRW homogeneous model can easily be made by approximating the induced metric (11) by

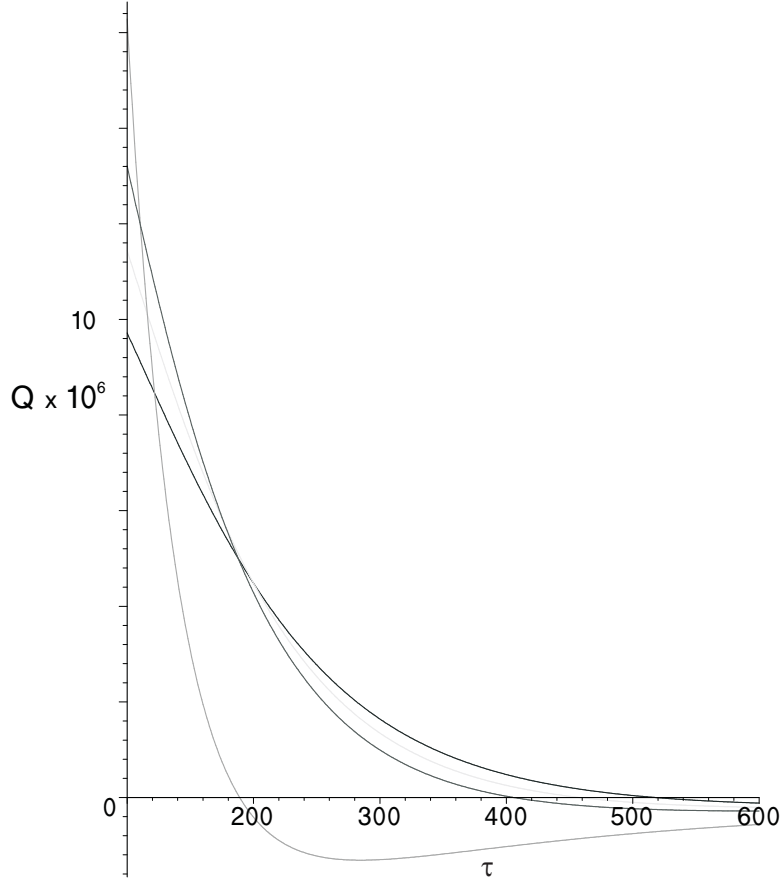
$$ds^2 \simeq ((t - t_0)^2/2 + \rho_0^2/6 + r_0)^2 \left( -dt^2 + d\varrho^2 + \varrho^2 d\Omega_2^2 \right), \quad (34)$$

around some fixed value of the radial distance  $\rho = \rho_0$  with  $\rho = \rho_0 + \varrho$  and  $(t - t_0) \gg \varrho$ . Then in “synchronous time”  $d\tau = ((t - t_0)^2/2 + \rho_0^2/6 + r_0)dt$  one can rewrite the metric as a FRW-like one

$$ds^2 \simeq -d\tau^2 + a(\tau)^2 \left( d\varrho^2 + \varrho^2 d\Omega_2^2 \right). \quad (35)$$

The expression for  $a(\tau)$  is rather involved so instead of giving its explicit form, Fig.(1) plots the acceleration parameter  $\frac{\partial^2 a}{\partial \tau^2} = \partial_\tau \theta + 1/3\theta^2$ , where  $\theta$  is calculated for time-like





**Figure 1.** “FRW-like” expansion  $Q = \ddot{a}/a$ . Units of  $AdS$  length, i.e.  $L = 1$ . Decreasing curves close to  $\tau = 0$  correspond to  $\rho = 10, 200, 220, 240$ . Values used,  $t_0 = 0$ ,  $r_0 = 1$  and  $\mu = 10^{-4}$ .

geodesic observers. At large proper time, the asymptotic expansion of the acceleration is

$$\frac{\ddot{a}}{a} \sim -\frac{2}{9}\tau^{-2}, \quad (36)$$

which is the acceleration of a dust-dominated Universe in a FRW background, as we shall now show.

Considered as a model for inflation, our model has certain advantages. In particular, a mirage-driven inflationary phase naturally ends when inhomogeneities are diluted enough by the inflation. Indeed, when  $\mu t^2$  dominates  $\mu \rho^2/6 + r_0$  in  $r$ , the metric becomes approximately homogeneous

$$ds^2 \simeq \frac{\mu^2}{4L^2} t^4 (-dt^2 + dx_3^2). \quad (37)$$

We can consider  $t$  as a conformal time and define the synchronous time  $\tau = \mu t^3/6L$  so that

$$ds^2 \simeq -d\tau^2 + \left(\frac{\tau}{\tau_0}\right)^{4/3} dx_3^2, \quad (38)$$

where  $\tau_0 = 3\sqrt{2L/\mu}$ . At late time the spacetime looks like FRW with a scale factor  $a(\tau) \sim \tau^{2/3}$ , as expected in a dust-dominated Universe. Moreover, the inhomogeneous mirage acceleration provides a dark matter component left over from an early-time inflationary phase.

#### 4. Homogeneity bounds

Mirage inhomogeneities may also account for the observed accelerating expansion of the Universe deduced from Type SNIa supernova observations and CMB WMAP data [5]. In this case, it would be enough for us to be at a distance from the center such that  $\rho > t - t_0 + \frac{r_0}{\mu(t-t_0)}$  to have acceleration of the light cone, thus explaining the observed cosmic acceleration. However there are constraints coming from the homogeneity bounds of the Universe.

The first one is on the homogeneity of the CMB spectra. In standard cosmology, an initial big bang-like singularity prevents very distant objects from having been in causal contact in the far past. In the present model, however, there is no initial singularity. It is thus allowed to consider a homogeneous radiation put on top of an inhomogeneous spacetime. Our model should be compared to tests on homogeneity at small scales. In particular, the Luminous Red Galaxies (LRG) data [15] of the Sloan Digital Sky Survey [16] can be used as a measure of the homogeneity of the Universe. The analysis suggests that the Universe is homogeneous for comoving distances of the order of  $R \sim 70h^{-1}\text{Mpc}$ . It actually reveals that our universe is homogeneous from around 100 Mpc up to distances of 140 Mpc [17],[18]. This means that within this shell, the number of luminous objects in a given volume  $V$  of a box of size  $\ell$  scales like  $\ell^3$ . Possible inhomogeneities are thus restricted and a comparison of the present model with observational data is welcome. So far, the model considered was empty of real matter. We consider the galaxies to be dust and would like to see that the mirage inhomogeneities are not inconsistent with the observed homogeneity of the Universe. In effect, since for dust the energy contained in a given volume will grow with the volume, we can roughly say that the number of object in the volume is proportional to the volume itself. Therefore, our Universe will look like homogeneous if the three-volume scales like the distance to the cube up to 140 Mpc. For this, we need to write down the induced metric (11) in comoving coordinates. This is not an easy task but we may find an approximate answer. First we need to find the coordinate transformation from  $(t, \rho, \theta, \phi)$  to  $(\tau, R, \theta, \phi)$  such that the 4-velocity  $u^\alpha = -\delta^\alpha_\tau$  where  $\tau$  is the proper time. So we impose:

$$-1 = u^\tau = \dot{\tau}u^t + \tau'u^\rho, \quad (39)$$

and

$$0 = u^R = \dot{R}u^t + R'u^\rho. \quad (40)$$

As before we will use the large- $\beta$  approximation. In particular, since we are interested in our position in the Universe, we will consider  $\beta \sim \rho^2/3$  in (22).

Equations (39,40) are solved by

$$R = \frac{\mu}{3}(t - t_0)^2 + \frac{\mu}{6}\rho^2 \simeq \frac{\mu}{6}\rho^2, \quad \tau = \frac{2}{3L}(t - t_0)\left[R + \frac{\rho^2\mu}{12}\right], \quad (41)$$

which can be inverted, giving

$$\rho = \sqrt{\frac{6R}{\mu}}, \quad t = L\frac{\tau}{R} + t_0. \quad (42)$$

In  $(\tau, R, \theta, \phi)$  coordinates, the metric is

$$\begin{aligned} g_{TT} &= -1, \\ g_{RR} &= g_{\rho\rho} \left[ (\partial_R \rho)^2 - (\partial_R t)^2 \right] \simeq \frac{3}{2} \frac{R}{\mu L^2}, \\ g_{RT} &= g_{tt} [\partial_R t \partial_\tau t - \partial_R \rho \partial_\tau \rho] \simeq \frac{\tau}{R}, \end{aligned} \quad (43)$$

so that the line element reads

$$ds^2 = -d\tau^2 + \frac{3}{2} \frac{R}{\mu L^2} dR^2 + 2 \frac{\tau}{R} dR d\tau + \frac{6R^3}{\mu L^2} d\Omega_2^2. \quad (44)$$

In order to calculate the three volume in a box around a given point, we introduce a new radial variable  $\xi$  so that the circumference of the circles in  $d\Omega_2^2$  is  $2\pi\xi$

$$\xi = \sqrt{\frac{6}{\mu L^2}} R^{3/2}, \quad (45)$$

in terms of which (44) becomes

$$ds^2 = -d\tau^2 + \frac{1}{9} d\xi^2 + \frac{4\tau}{3\xi} d\xi d\tau + \xi^2 d\Omega_2^2. \quad (46)$$

To calculate the volume of a box centered at our position it is useful to use 'cartesian' coordinates so that  $d\xi^2 + \xi^2 d\Omega_2^2 = dx^2 + dy^2 + dz^2$ , and  $\xi^2 = x^2 + y^2 + z^2$ . The line element becomes:

$$\begin{aligned} ds^2 &= -d\tau^2 + \left(1 - \frac{8x^2}{9\xi^2}\right) dx^2 + \left(1 - \frac{8y^2}{9\xi^2}\right) dy^2 + \left(1 - \frac{8z^2}{9\xi^2}\right) dz^2 + \frac{4x\tau}{3\xi^2} dx d\tau \\ &\quad + \frac{4y\tau}{3\xi^2} dy d\tau + \frac{4z\tau}{3\xi^2} dz d\tau - \frac{16}{9} \frac{dx dy}{\xi^2} - \frac{16}{9} \frac{dx dz}{\xi^2} - \frac{16}{9} \frac{dy dz}{\xi^2}. \end{aligned} \quad (47)$$

For simplicity, we consider our spatial position in the Universe to be at the point  $(x_0, x_0, x_0)$ , where, as discussed before,  $x_0$  is large with respect to any other length considered. There is nothing particular about this point and it has been chosen for computational simplicity only. The volume of a box of coordinate side size  $2x_1$ , centered at  $(x_0, x_0, x_0)$  can now be estimated. We first need to find the proper length of the box. The proper length of side of the box is given by

$$\ell = \int_{-x_1}^{x_1} v_\alpha dx^\alpha, \quad (48)$$

where the 4-vector  $v_\alpha$  is tangent to one of the box's edge and satisfies  $v_\alpha u^\alpha = 0$  and  $v_\alpha v^\alpha = 1$ .

The volume  $V$  of the box for large  $x_0$  is then:

$$V \propto \ell^3 \left(1 - \frac{8}{\sqrt{33}} \frac{\ell}{x_0}\right) = \ell^3 \left(1 - \frac{4\ell}{\sqrt{11}\sqrt{2}} \sqrt{\Lambda}\right), \quad (49)$$

where, tracing back the coordinate transformations, we used

$$x_0 = \frac{\mu\rho_0^3}{6\sqrt{3}L} \simeq 2\sqrt{\frac{2}{3}}\Lambda^{-1/2}, \quad (50)$$

where we used (29) for the last step.

To compare with observations we take  $\Lambda^{-1/2} = 2.6 \times 10^3$  Mpc [5]. We check that the fractal dimension of our Universe up to 140 Mpc away from the observer, and with a cut off at the scale of inhomogeneities taken to be at 100 Mpc [17], is within the observed homogeneity bound. Denoting the 100 Mpc cut-off values for the volume and length by  $V_c$  and  $l_c$  respectively, the fractal dimension  $D$  we are looking for is defined by:

$$D = \frac{\log \frac{V}{V_c}}{\log \frac{l}{l_c}} \quad (51)$$

which we find to be  $D = 3 \pm 0.03$ , i.e. within the observed bound [17].

## 5. Conclusions

In this paper, we introduced an inhomogeneous type of solution of a model of a D3-brane moving in an  $AdS_5 \times S_5$  bulk. We assumed that all matter contributions are subdominant to the geometrical mirage energy contribution. Our brane is inhomogeneous and although from the bulk viewpoint there is no special point, a center of the Universe must be introduced. This center appears due to the inhomogeneity, it is not singular and the Universe is isotropic around this point. The cosmological evolution does not have an initial singularity and does not have a cosmological singularity. This prevents us from having a horizon problem. We found that the brane is initially accelerated with increasing magnitude as the distance from the center decreases. The acceleration naturally ends in a FRW-like dust dominated Universe at late time as the distance from the center becomes large. The mirage matter can then be thought of as a component of the observed dark matter.

We may also consider this model for explaining the present-day observed cosmological acceleration. In this case, the cosmological constant can be understood as being geometrical, since it is inversely proportional to the distance to the center of the Universe. A small cosmological constant simply means that our position is far from the center.

It is possible that the introduction of matter can connect the early and late time accelerations with a decelerated era driven by matter. However, to explain how this mechanism might happen is beyond the scope of the present paper.

A potential problem in these type of models, is that objects around us (like galaxies) look homogeneously distributed. Analysis of observational data suggests that we live in a homogeneous Universe up to a scale of  $\sim 140$  Mpc [17],[18] (Below 100 Mpc, the inhomogeneities correspond to the coarse graining of the energy density). This constrains the allowed inhomogeneity for a late-time mirage acceleration and does not

rule it out. It is also interesting to note that the observed cosmological constant introduces a scale at which the inhomogeneities of our model could in principle be observed. More precisely, our model predicts inhomogeneities at scales just above the observationally observable ones. It is intriguing that the very small inhomogeneities of the CMB spectra from the WMAP data are actually orthogonal to the ecliptic plane [21]. It has been suggested that this phenomena can be explained introducing inhomogeneities in the Universe [19]. However, we need to test our model against the full CMB spectrum. This could probably be done by considering the effective four-dimensional Einstein equations obtained by re-writing the geodesic equations of the brane in terms of standard General Relativity with matter sources. In this case, if these sources can be re-interpreted as coupled scalar fields, at least under some approximations, it might be possible to compare our model with standard cosmology using the techniques developed in [20] for the study of the CMB power spectrum and bispectrum on inhomogeneous background geometries sourced by coupled scalar fields  $\parallel$ . This is, however, postponed for future research.

Our model does not consider matter contributions. In order to be tested against BBN nucleosynthesis it must be implemented with matter. Since early time matter and BBN constraints only deal with radiation, one could imagine a consistent model introducing a small contribution of Maxwell fields on the brane and then study perturbations on it. We believe this will not spoil the acceleration as the Maxwell fields will always be sub-dominant. However this have to be tested with numerical calculations and is therefore beyond the scope of this paper. An additional constraint would be to check our model against the scale invariant perturbation observed in the CMB, although it is not clear what a perturbation would mean in this case as there is no homogeneous background. It should be noted that another point, connected with the brane-matter contributions, which should be checked is the flatness of the Universe. However, as long as we have not introduce any matter we cannot answer this since an estimate of the total energy density versus the critical one is lacking in this case.

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